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Ionized-Impurity Scattering Mobility of Electrons in Silicon

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A quantitative, partly empirical formula for the ionized-impurity scattering mobility of electrons in silicon, derived earlier by Long and Myers from an analysis of resistivity and Hall effect data, is compared with a theoretical formula of Samoilovich, Korenblit, and Dakhovskii for this mobility, which enables one for the first time to calculate its magnitude from the known conduction band parameters of silicon. Very good quantitative agreement between the two formulas is found.

THE first detailed calculations of anisotropic electron scattering by ionized impurities in materials like n-type silicon and germanium have recently been published by Samoilovich, Korenblit, and Dakhovskii.[1] Their calculations proceed from essentially the same assumptions on which well-known earlier theories of impurity scattering were based, but they have extended the situation to account for anisotropy in the scattering.[1,2] The explicitly stated assumptions in the SKD theory are:

(a) The energy ϵ vs wave number k relationship has the anisotropic form

$$\epsilon=\sum_{i=1}^{3}\frac{\hbar^2k_i^2}{2m_i},\quad m_1=m_2<m_3. \tag{1}$$

In n-type silicon the effective mass parameters determined by cyclotron resonance are $m_1=(0.192\pm0.001)m_0$ and $m_3=(0.90\pm0.02)m_0$, where m_0 is the free-electron mass.[3] The m_1 value is well established, but other cyclotron resonance experiments have given $m_3=(0.98\pm0.04)m_0$, although the $m_3=0.90m_0$ value is probably the better. The mass anisotropy is the cause of the scattering anisotropy according to the SKD calculations.

(b) No magnetic field exists (this actually means that the field must not affect the scattering probability).

(c) The impurity potential V has the form

$$V=(q^2/\kappa r)e^{-r/a}, \tag{2}$$

where κ is the dielectric constant ($\kappa=11.7$ for Si), q the electronic charge, r a radius from the ionized impurity center, and a the screening radius.

(d) The Born approximation applies.

The SKD results are expressed in terms of relaxation-time tensor components $\tau_1=\tau_2<\tau_3$, where the subscripts refer to the directions in the constant-energy spheroids defined by Eq. (1).

The rigorous validity of some of the explicit and implicit assumptions in the theory is questionable for doping levels in silicon and germanium high enough to give easily observable impurity scattering,[4] and yet the earlier mobility formulas resulting from these assumptions (e.g., the Brooks-Herring formula) have been shown to give a very good functional description of mobility data, at least in n-type silicon.[5,6] In applying the earlier formulas to the data, however, the effective mass appearing in them had to be treated as a parameter to be determined empirically, since it was not clear exactly how to calculate its value from the known anisotropic mass parameters. The SKD results now provide a means for making such a calculation, and the purpose of this note is to do so for n-type silicon and then to compare the resulting impurity scattering mobility formula with the quantitative formula determined empirically in previous studies.[5,6]

In earlier work we found by analysis of resistivity and Hall effect vs temperature data that the impurity scattering electron mobility μ_I in Si samples with doping levels up to nearly 10^{16} atoms per cm^3 is well described

[1] A. G. Samoilovich, I. Ya. Korenblit, and I. V. Dakhovskii, Soviet Phys.—Doklady 6, 606 (1962); referred to as SKD in the text.
[2] A. G. Samoilovich, I. Ya. Korenblit, I. V. Dakhovskii, and V. D. Iskra, Soviet Phys.—Solid State 3, 2385 (1962).
[3] C. J. Rauch, J. J. Stickler, H. J. Zeiger, and G. S. Heller, Phys. Rev. Letters 4, 64 (1960).
[4] See, for example, C. Herring, T. H. Geballe, and J. E. Kunzler, Bell System Tech. J. 38, 657 (1959).
[5] D. Long and J. Myers, Phys. Rev. 115, 1107 (1959).
[6] D. Long and J. Myers, Phys. Rev. 120, 39 (1960). The Appendix of this paper gives a corrected quantitative expression for the ionized-impurity scattering mobility of electrons in Si which is believed to be more accurate than the expression given in reference 5.

by the formula[5,6]

$$\mu_I \cong \frac{7.0\times10^{17}T^{3/2}}{N_I(\ln b-1)} \text{ cm}^2/\text{V-sec}, \quad (3)$$

where T is the absolute temperature (which ranged from 30 up to 100°K), N_I is the total ionized concentration of donors N_D and acceptors N_A, and

$$b=4.7\times10^{14}T^2/n', \quad (4)$$

where

$$n'=n+(n+N_A)[1-(n+N_A)/N_D], \quad (5)$$

and n is the conduction band carrier density. The numerical coefficient in Eq. (3) is believed to have a *maximum* uncertainty of ±25%.

Equation (3) does not allow explicitly for any anisotropy in the scattering, but rather implicitly contains a sort of averaged isotropic approximation to the anisotropic impurity scattering[6] for the doping levels and temperatures being considered. Mobilities are, of course, isotropic, and the numerical scalar coefficient in Eq. (3) was derived empirically simply by determining what its value must be to give the observed reduction in mobility from its value in the presence of only lattice scattering.[5] The rigorously correct way to compare the SKD results with experiment would be to re-analyze the mobility data[5] taking proper account of the anisotropies of both lattice and impurity scattering by calculating each observed mobility from the expression[7]

$$\mu=(q/3)(2\langle\tau_1\rangle/m_1+\langle\tau_3\rangle/m_3), \quad (6)$$

where τ_1 and τ_3 are determined for each case by adding reciprocals of the lattice and impurity scattering relaxation time components,[6,7] and the angular brackets represent Maxwellian averages. Fortunately, however, no such laborious procedure is necessary, because the magnitude of the observed mobility proves to be quite insensitive to the scattering anisotropies in the cases of interest here. Very little error is involved if one simply ignores the anisotropy and assumes arbitrarily that τ_3 is the same as τ_1. This approach is satisfactory mainly because (1) $m_3\gg m_1$, so that the great majority of the conduction is in the 1 and 2 directions, and because (2) the known anisotropies of lattice[6] and impurity[1,6] scattering are opposite and, therefore, tend to cancel. We can then compare Eq. (3) directly with the SKD results by assuming for this purpose that the impurity scattering is described by a single scalar relaxation time equal to τ_1. Even in the worst case, that corresponding to the most heavily doped samples of reference 5 at 30°K, the error due to this procedure can be shown to be only about 10%. This error is much less than the experimental uncertainty associated with the numerical coefficient in Eq. (3), and in all other cases the error is smaller than 10% and generally considerably smaller.

The SKD formula for the relaxation time τ_1 is[8]

$$\tau_1=\frac{8\kappa^2m_1\beta^3\epsilon^{3/2}}{3\pi N_I q^4 2m_3}(1+g_1)f(\beta,b), \quad (7)$$

where $\beta^2=(m_3-m_1)/m_1$, $(1+g_1)$ is a very weak function of b which is equal to about 1.1 for any doping level and temperature of interest here, and the complicated function $f(\beta,b)$ is defined in references 1 and 2. Ignoring any difference between τ_1 and τ_3 for the reasons given above, the impurity scattering mobility will then be[7]

$$\mu_I=(q\langle\tau_1\rangle/3)(2/m_1+1/m_3), \quad (8)$$

where $\langle\tau_1\rangle$ is the Maxwellian average of τ_1. Evaluation of Eq. (8) for n-type silicon with the use of Eq. (7) gives[8]

$$\mu_I\cong\frac{7.5\times10^{17}T^{3/2}}{N_I(\ln b-1.20)} \text{ cm}^2/\text{V-sec}, \quad (9)$$

assuming that $b\geq3\times10^2$, which was always true in the experimental work from which Eq. (3) was derived.[5,6] Note that if m_3 were $0.98m_0$ instead of $0.90m_0$, the numerical coefficient in Eq. (9) would drop slightly to 7.3.

We see that Eqs. (9) and (3) agree very well, especially since at the concentrations and temperatures of interest one always has ln $b\gg1$. Thus, the *magnitude* of the ionized impurity scattering mobility is predicted quite accurately for n-type silicon by the SKD theory, at least for impurity concentrations below 10^{16} cm^{-3} temperatures above 30°K. The *anisotropy* of the impurity scattering in n-type silicon predicted by the SKD theory also agrees well with experimental results; viz., data on the saturation longitudinal magnetoresistance, an analysis of which will be reported soon.[9]

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[7] C. Herring and E. Vogt, Phys. Rev. **101**, 944 (1956).

[8] Note that our b can easily be shown to be identical to SKD's γ^{-2} except for a difference of a numerical factor, and that we have simply re-expressed SKD's results in terms of b instead of γ^{-2}. We have also let $\epsilon=3kT$ in evaluating γ^{-2}, as is conventional.

[9] L. J. Neuringer and W. J. Little, in *Proceedings of the 1962 International Conference on Semiconductor Physics* (to be published); L. J. Neuringer and D. Long (to be published).